European Xtramile Centre of African Studies (EXCAS)

EXCAS Working Paper

WP/19/092

Dependence risk analysis in energy, agricultural and precious metals commodities: A pair vine copula approach

Forthcoming: Journal of Applied Economics

Satish Kumar
IBS Hyderabad, (ICFAI Foundation for Higher Education), India

Aviral K. Tiwari
Montpellier Business School, Montpellier, France

Ibrahim D. Raheem
The European Xtramile Centre of African Studies (EXCAS), Liège, Belgium

Qiang Ji
Center for Energy and Environmental Policy Research, Institutes of Science and Development, Chinese Academy of Sciences, Beijing 100190, China and,
School of Public Policy and Management, University of Chinese Academy of Sciences, Beijing 100049, China

Research Department

Dependence risk analysis in energy, agricultural and precious metals commodities: A pair vine copula approach

Satish Kumar, Aviral K. Tiwari, Ibrahim D. Raheem & Qiang Ji

January 2019

Abstract

We apply pair vine copulas, specifically the C-vine and R-vine copulas, to examine the conditional multivariate dependence pattern/structure and R-vine copula-based value-at-risk (VaR) to assess financial portfolio risk. We examine the co-dependencies of 13 major commodity markets (which include three energy commodities, six agricultural commodities and four precious metals prices) from 2 January 2003 to 19 December 2016. Dividing our sample into three sub-periods, namely pre-GFC, GFC and post-GFC, we find that the dependencies among commodities undergo changes in a complex manner, changing in different financial conditions, and that the Student-*t* copula appears on the maximum number of occasions, especially during the GFC period, signifying the existence of fatter tails in the distributions of returns. We further show that the co-dependencies computed using R-vine copulas are best suited to compute the portfolio VaR during the considered time period.

Keywords: R-vine; VaR; Dependence structure; Tree structure; Commodity markets

1. Introduction

In the last few years, the linkages between energy commodities, agricultural commodities and precious metals have increased thanks to the financialization of commodity markets (Tang and Xiong, 2012). For investors to manage risk in their portfolios effectively, it is crucial to measure the dependence structure of commodities (Liu et al., 2017). Ever since the global financial crisis (GFC), researchers and practitioners have shown greater curiosity in regards to examining the dynamics of dependence among commodity markets by applying new techniques for dependence estimation and portfolio optimisation. In this context, an accurate evaluation and analysis of the dependence structure and dependence risk of energy, agricultural and precious metals commodities have practical implications for both investors and policymakers, given the economic linkages and dependence relationships among these major commodities (Hernandez, 2015).

The dependence structure across different commodity markets has received much attention in the field of commodity financialization and risk management. The main methodologies applied to model the co-movement and risk dependence among commodities can be divided into the cointegration and correlation measure (Nazlioglu and Soytas, 2012; Nicola et al., 2016), the autoregressive distributive lag (ARDL)model (Bouri et al., 2018), the structural vector autoregression (SVAR) model (Fernandez-Perez et al., 2016), the generalized autoregressive conditional heteroskedasticity (GARCH)-type model (Ji and Fan, 2012; Mensi et al., 2014; Kang et al., 2017; Fernandez-Diaz and Morley, 2018), the network model (Ji and Fan, 2016a, b; Ji et al., 2018a), the wavelet model (Lahmiri et al., 2017; Antonakakis et al., 2018; Mensi et al., 2018) and the conditional value-at-risk measure (Shahzad et al., 2018; Ji et al., 2018b).

However, the dependence structure of commodities might be complicated and time-varying and show nonlinear behaviour, thereby requiring the employment of advanced models which could effectively interpret their dependence and joint distributions. Pair vine copula modelling has been one such sophisticated technique and has received considerable attention in financial economics, especially in the last decade. It has been found to outperform the alternative modelling techniques used to estimate the dependence structure of various asset classes (Heinen and Valdesogo, 2009; Dismman et al., 2013). Further, it overcomes the restrictive characteristics of the bivariate copulas and traditional measures of correlation (Brechmann and Czado, 2013; Ji et al., 2018c). Vine copulas offer better flexibility compared to standard copula models in that they allow the modelling of complex dependency structure which may be analysed in the form of tree structure.

Our study makes use of canonical vine (C-vine) and regular vine (R-vine) copula models to examine the dependence structure and risk characteristics of 13 commodities at times of extreme financial stress and their usefulness in capturing the changes in their co-dependencies. The data comprise six agricultural commodities (corn, soybean oil [SO], soybean meal [SM], oats, soybeans and wheat), four precious metals commodities (gold, silver, palladium [PD] and platinum [PT]) and three energy commodities (West Texas Intermediate [WTI], heating oil [HO] and natural gas [NG]). Our study is motivated to analyse the commodity markets using the pair vine copula and portfolio optimisation approach because investors are increasingly attracted to these markets to diversify their portfolios (Hernandez, 2014). We analyse the data in three different sub-periods: pre-GFC, GFC and post-GFC. To further highlight the capabilities of our flexible modelling framework, we also employ R-vine copulas to quantify the value-at-risk (VaR) for an equally weighted portfolio of our considered commodities.

We contribute to the literature in many ways. First, we identify the risk characteristics of commodity portfolios and changes in their conditional dependence structure across three financial periods, incorporating the GFC. Second, we select the vine copula models that capture the multivariate risk dependence most effectively. To do so, we employ a copula technique to account for the complicated analysis of the portfolio's dependence structure and the inherent changes over different time periods. Finally, we demonstrate the calculation of portfolio VaR on the basis of these dependency measures. Our results indicate that the dependencies change in a complex way and that the Student-*t* copula appears on the maximum number of occasions, especially during the GFC period, signifying the existence of fatter tails in the distributions of returns. For the purpose of the comparisons, we also used GARCH-based portfolio VaR, and the comparisons show that the R-vine copula-based portfolio VaR outperforms. Finally, we also provide the portfolio analysis (based on returns) using efficiency frontier in two scenarios: when short sale is or is not allowed.

The rest of the paper is organised as follows. The next section describes the methodology in detail, while Section 3 presents the data. Section 4 explains the main empirical results. Section 5 presents the application of R-vine copulas to the VaR framework. Finally, Section 6 concludes the article.

2. Methodology

2.1.Background Description

Copula analyses and their applications have their origin in the statistical and mathematical fields. However, the integration and inter linkages among fields have led to the applicability of the models beyond their origin. Copulas can be attributed to Frechet (1957) and Sklar (1996) and gained popularity in the late 1990s. In brief, copulas can be described as functions that connect the multivariate and marginal distribution functions of any dimension by utilising all relevant information about the dependence structure between variables that are considered to be random.

Thus, the copula is one of the few methods that allow for greater flexibility in analysing multivariate and marginal distributions. The earlier forms of copulas are bivariate in nature (parametric and non-parametric). However, complexity arises when modelling more than two dimensions. This problem was first resolved by Joe (1997), with the formulation of multivariate dimension based on pair-wise copulas. Also, Bedford and Cooke (2002) introduced the concept of using diagrams to decompose multivariate copulas to pair-copulas. Thus, the work of Joe and Bedford and Cooke can be considered the genesis of paircopulas or vinecopulas.

2.2.Basic Concept

Copulas are functions that join multivariate distributions to their marginal distributions. These functions have uniform one-dimensional margins with an interval of [0,1], which have invariant monotonic increasing transformations of the marginal (Nelsen, 2006). The functionality of copulas assumes the existence of a vector of X random variables with marginal distribution functions of $F_i(X_i), i = 1, 2, \dots, d.$ The set of transformation $U_i = F_i(X_i)$ defines a dependent and uniformly distributed vector of random variables$\boldsymbol{U} = (U_1, \dots, U_d)$ on $[0,1]^d$. Nelson assumes that the joint distribution function of X can be expressed in the form below if the functions of $F_i(X_i)$ are continuous in nature:

$$F(x) = C\big(F_1(x_i), \dots, F_d(x_d)\big) = C(U_1, \dots, U_d), \tag{1}$$

where C(U) is the copula of the distribution, C:$[0,1]^d \to [0,1]$ and U = $(U_1, \dots, U_d)$. The copula c could also be likened to a joint distribution function of vector U. Equation (1) above is the Sklar's theorem. The equation could be expanded by defining copula C(U) as:

$$C(u) = F\left(F_1^{-1}(u_1), \dots ,\ F_d^{-1}(u_d)\right), \tag{2}$$

and the accompanying copula density is given as:

$$c(u) = {\partial^d C(U_1, \dots , U_d)} \Big/ {\partial(U_1, \dots , U_d)}. \tag{3}$$

Lebrun and Dutfoy (2009a) define the joint probability density function of X, $f_X(x_d) = f_X(x_1, \dots , x_d)$ as:

$$f_X(x_1, \dots , x_d) = c\{F_1(x_1), \dots , F_d(x)\} \prod_{i=1}^{d} f(x_1), \tag{4}$$

where $f_i(x_i)$ is the marginal probability density function of X_i. Equation (4) combines the marginal distributions and copula density, which contains all relevant information about the dependence structure of the random variables. Genest et al. (2009) and Lebrun and Dutfoy (2009b) calculate the conditional marginal distributions of $\boldsymbol{X}$ as:

$$F_{i|1,\dots,j-1}(x_i|\ x_i, \dots , x_{i-1}) = \ C_{i|1,\dots i-1}(u_i|\ u_i, \dots , u_{i-1}), \tag{5}$$

where

$$C_{i|1,\dots i-1}(u_i|\ u_i, \dots , u_{i-1}) = \left. \frac{\partial^{i-1} C(U_1,\dots,U_i,1,\dots,1)}{\partial(U_1,\dots,U_i)} \middle/ \frac{\partial^{i-1} C(U_1,\dots,U_i,U_{i-1},1,\dots,1)}{\partial(U_1,\dots,U_i)} \right. . \tag{6}$$

Equation (6) can also be considered for the bivariate case, with $u_1 = u$ and $u_2 = v$:

$$F_{X2|X1}\,(x_2|\ x_!) = C(v|u) = {\partial C(u, v)} \Big/ {\partial u}. \tag{7}$$

If a random sample of size n having a corresponding vector of variables X, i.e. $x_1 = (x_{i1}, \dots , x_{id})(i = 1, \dots , n)$ is given, then, according to Genest et al. (1995) and Genest and Favre (2007),the estimated copula is defined as:

$$C_n(u) = \frac{1}{n} \sum_{i=1}^{n} 1(S_{i1} \le\ U_1, \dots , S_{id} \le\ U_d), \tag{8}$$

where$S_{ij} = r_{ir}/(n+1)$ are the pseudo-observations, and r_{ir} are the ranks associated with the sample. The pseudo-observations can be grouped into vector $S_i = (S_{i1}, \dots, S_{id})$.

2.3.Vine Copulas

Aas et al. (2006) defined vine copulas as any pair-copula construction. The joint probability density functions if X, $f_x(x) = f_x(X_1, \dots, X_d)$ can be expressed in terms of conditional density functions as:

$$f(x) = f_1(x_1) f_{2|1}(x_2|x_1) f_{3|1,2}(x_3|x_1, x_2), \dots, f_{d|1,\dots,d-1}(x_d|x_1, \dots, x_{d-1}). \quad (9)$$

It should be noted that Equation(9) is not a representation of a unique decomposition; it is one of the many ways in which the function $f_X(x_d)$ can be decomposed. However, the bivariate decomposition is represented as:

$$f_{12}(x_1, x_2) = C_{12}\big(F_1(x_1), F_2(x_2)\big) f_1(x_1) f_2(x_2), \quad (10)$$

where C_{12} is the copula density between variable X_1 and X_2. Subjecting Equation (10) to conditional probability, we have:

$$f_{2|1}(x_1|\, x_2) = C_{12}\big(F_1(x_1), F_2(x_2)\big) f_2(x_2). \quad (11)$$

Equation (11) can be extended to a three random variable dimension such that it is expressed as:

$$f_{3|1,2}\big(x_3|\, x_{1,} x_2\big) = C_{23|1}\left(F_{2|1}(x_2|x_1),\ F_{3|1}(x_3|x_1)\right) f_{3|1}(x_3|x_1). \quad (12)$$

The implicit assumption in Equation (12)is that $C_{23|1}$ is independent of X_1, which allows for flexible modelling as a result of it being considered a good approximation (Brechmann, 2010). However, Equation (12) could be expressed in other forms. For instance, an alternative specification of the form is:

$$f_{3|1,2}\big(x_3|\, x_{1,} x_2\big) = C_{13|2}\left(F_{1|2}(x_1|x_2),\ F_{3|2}(x_3|x_2)\right) f_{3|2}(x_3|x_2). \quad (13)$$

Equations (12) and (13) show that the conditional densities can be decomposed into appropriate pair copulas and marginal (conditional) densities. The possibility of further decomposition of Equation (13) is explored as illustrated in Equation (11), which thus leads to:

$$f_{3|1,2}(x_3|\ x_{1,}x_2) = C_{13|2}\left(F_{1|2}(x_1|x_2),\ F_{3|2}(x_3|x_2)\right) C_{23}\big(F_2(x_2), F_3(x_3)\big) f_3(x_3). \tag{14}$$

In a situation in which two pair copulas are involved (for example Equation (9)), each term can be decomposed into appropriate products of conditional pair copulas multiplied by the conditional densities using the formula below:

$$f(x|v) = c_{x,v_j|v_{-j}}\left(F(x|v_{-j}), F(v_j|v_{-j})\right)\ f(x|v_{-j}), \tag{15}$$

where v_j is an arbitrary selected component of a vector v, v_j is v excluding such component and $c_{x,v_j|v_{-j}}$ is the conditional bivariate copula. The paircopula that involves marginal conditional distribution can also be obtained using a recursive approach based on the formula below:

$$F(x|v) =\ \partial c_{x,v_j|v_{-j}}\left(F(x|v_{-j}), F(v_j|v_{-j})\right), F(x|v_{-j})/\partial F(x|v_{-j}). \tag{16}$$

There is a special case of univariate *V*, which can be expressed as:

$$F(x|v) =\ \partial c_{x,v}\big(F(x), F(v)\big)/\partial F(v). \tag{17}$$

Following Aas et al. (2009), the C-vines copula is expressed as:

$F_{i|1,\dots,i-1}(x_i|x_1, \dots, x_{i-1}) =$

$\partial C_{i,i-1|1,,\dots,1-2}\big(F(x_i|x_1, \dots, x_{i-2}), F(x_{i-1}|x_1, \dots, x_{i-2})\big),/\partial F\big((x_i|x_{1-1}|x_1, \dots, x_{i-2})\big)$(18)

2.4.Regular-Vine Copulas

The main criticism of earlier versions of the pair copula is their inability to solve complicated models. As such, Dissmann (2010) suggested a possible way of solving this problem by constructing a regular vine using a diagram algorithm. In order words, the regular vine serves as an improvement on the variant of prior works due to the flexible nature of the former, as it can model a wide range of complicated dependencies. However, R-vine has also been criticised based on the curse of dimensionality. The computational effort required to estimate the model also grows exponentially with the dimension.

A section of the literature has identified that among the panaceas to this problem is either to simplify or truncate the model. Allen et al. (2007) define the truncation of a regular vine at level K as a situation in which any pair copulas that are equal to or larger than K are replaced with independent copulas. The independent copulas are regarded as Gaussian copulas, which are: (i) easier to specify that other variants of copulas and (ii) easier to interpret in terms of the correlation parameter. Vuoung (1989) proposed that the statistics that could be used are AIC-, BIC- and likelihood-ratio-based tests. The general specification of a regular-vine is expressed below:

$$f\left(x_1, \dots, x_d = \left[\prod_{k=1}^{d} f_x(x_k)\right] * \left[\prod_{i=1}^{d-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)} \left(F\left(x_{j(e)}|x_{D(e)}\right)\right), \left(F\left(x_{k(e)}|x_{D(e)}\right)\right)\right]\right) \quad (19)$$

3. Data

We analyse the daily returns of six agricultural commodities – corn, SO, SM, oats, soybeans and wheat; four precious metals – gold, silver, PD and PT; and three energy commodities – WTI, HO and NG– from 2 January 2003 to 19 December 2016. The data were accessed from the Bloomberg website. We later divided our data into three sub-periods, pre-GFC (January 2003 to June 2007), GFC (July 2007 to August 2009) and post-GFC (September 2009 to December 2016). Table 1 illustrates the descriptive statistics for the 13 commodity markets for the full sample and three sub-periods. It is not surprising that the mean returns are positive for all commodities in the pre-crisis period and negative during the GFC period (except for gold, silver, SM and soybeans).

Table 1: Descriptive statistics for all commodities

	Mean	SD	Skewness	Kurtosis	J-B	N
Full Sample						
Corn	0.011	1.946	-0.653	15.440	22876.9	3509
Gold	0.034	1.208	-0.375	7.752	3384.21	3509
HO	0.018	2.211	-0.251	6.887	2245.66	3509
NG	-0.012	3.300	0.633	7.645	3388.82	3509
Oats	0.003	2.279	-0.524	9.390	6129.94	3509
PD	0.030	2.095	-0.388	6.600	1983.38	3509
PT	0.012	1.455	-0.520	6.541	1991.40	3509
Silver	0.034	2.162	-0.874	9.431	6492.54	3509
SM	0.017	2.454	-1.773	54.084	383376	3509
SO	0.015	1.669	-0.495	15.004	21209.8	3509
SB	0.016	1.817	-1.453	27.739	90718.7	3509
Wheat	0.006	2.098	0.149	5.267	764.667	3509
WTI	0.014	2.433	0.103	7.019	2367.73	3509
Pre-GFC						
Corn	0.035	1.712	0.658	5.744	432.816	1122
Gold	0.056	1.113	-0.725	6.151	562.615	1122
HO	0.075	2.384	-0.131	4.417	97.0991	1122
NG	0.023	3.623	0.686	7.301	952.803	1122
Oats	0.023	2.188	-0.751	10.002	2397.78	1122
PD	0.038	2.186	-0.358	7.253	869.718	1122
PT	0.067	1.229	-0.430	4.973	216.561	1122
Silver	0.084	2.010	-1.246	10.990	3275.03	1122
SM	0.030	3.016	-1.780	66.048	186424	1122
SO	0.051	1.911	-1.071	22.481	17956.7	1122
SB	0.037	1.800	-2.494	37.654	57304.8	1122
Wheat	0.053	1.814	0.534	4.510	159.905	1122
WTI	0.071	2.156	-0.261	4.196	79.6145	1122
GFC						
Corn	-0.013	2.428	-0.291	3.694	18.65981	546
Gold	0.067	1.637	0.262	5.696	171.5821	546
HO	-0.027	2.761	-0.106	4.325	40.99189	546
NG	-0.150	3.400	0.169	4.774	74.18986	546
Oats	-0.043	2.405	-0.102	4.178	32.5211	546
PD	-0.044	2.532	-0.394	6.173	243.1024	546
PT	-0.007	2.184	-0.619	5.417	167.8183	546
Silver	0.030	2.745	-0.409	6.655	319.1952	546
SM	0.080	2.546	-1.382	10.722	1530.466	546
SO	-0.015	2.193	-0.081	4.242	35.66754	546
SB	0.037	2.663	-0.843	19.701	6410.401	546
Wheat	-0.039	2.870	-0.097	4.510	52.73616	546
WTI	-0.003	3.555	0.174	5.805	181.7269	546

Post-GFC						
Corn	0.007	1.919	-1.396	26.121	41559.0	1839
Gold	0.010	1.110	-0.755	8.909	2850.30	1839
HO	-0.003	1.897	-0.511	11.772	5976.52	1839
NG	0.010	3.053	0.758	8.852	2800.22	1839
Oats	0.006	2.296	-0.543	10.900	4872.96	1839
PD	0.047	1.886	-0.368	5.231	422.881	1839
PT	-0.016	1.304	-0.265	4.288	148.691	1839
Silver	0.004	2.053	-0.939	9.679	3688.67	1839
SM	-0.006	1.994	-1.694	17.430	16833.7	1839
SO	0.003	1.288	0.176	3.954	79.2659	1839
SB	0.000	1.479	-0.905	8.734	2770.12	1839
Wheat	-0.007	1.985	0.201	4.755	248.385	1839
WTI	-0.015	2.171	0.218	6.338	868.351	1839

Note: HO, NG, PD, PT, SM, SO and SB stand for heating oil, natural gas, palladium, platinum, soybean meal, soybean oil and soybeans, respectively. The data period is from 2 January 2003 to 19 December 2016.

The post-GFC returns are either positive or negative but less than the returns of the pre-GFC period. Most importantly, the standard deviation is higher in all commodity markets in the GFC period (except NG). The Jarque-Bera test rejects the null hypothesis of normality of daily returns for all commodities in all sub-periods. The returns are mostly negatively skewed; only gold, NG and WTI exhibit positive skewness during the GFC. In fact, NG exhibits positive skewness in the full sample as well as in the three sub-periods.

All the markets are also characterised by high kurtosis. Overall, the results in Table 1 show that the commodity returns in our sample are non-Gaussian and exhibit different skewness and kurtosis in different sub-periods. Therefore, copula models are best suited to capture these properties of the data having fat tails and changes in distributional characteristics.

4. Empirical Results

We present our results in two parts. In the first, we model the dependence structure of the considered commodities using the C-vine and R-vine copulas in three sub-periods, pre-GFC,

GFC and post- GFC. The second analysis pertains to the results from an empirical modelling of VaR using R-vine copulas for a portfolio constructed from the considered series.

4.1.Dependence Modelling using Vine Copula

As explained above, we have divided the data into three sub-periods of pre-GFC, GFC and post-GFC to carry the C-vine and R-vine dependence analysis for 13 commodities. To do so, we need suitably standardised marginal distributions for the commodity return series. We used the AR(1)-GARCH(1,1) as the base marginal model, but the final order of AR, and P & Q term GARCH model were selected based on AIC criteria. We extract the residuals and standardised them to obtain their marginals which were then used as inputs to the appropriate copula selection. We select the copula using the AIC criterion. The results described are for the pre-GFC period first, followed by the GFC and post-GFC periods.

In the next section, we shift our discussion to the more flexible R-vine copulas. We primarily focus on the results obtained from R-vine copulas; therefore, the results for C-vine copulas are presented in the appendices to conserve space. Moreover, the results for C-vine copula highlight their lesser flexibility since the same security number appears on most of the occasions across the rows (Refer to appendices). The R-vines, however, are more flexible in this regard, as will be shown later in the paper. We use a range of seven copulas for selection purposes using AIC as the criterion to choose from the following copulas: 1 = Gaussian copula, 2 = Student-*t* copula (*t*-copula), 3 = Clayton copula, 4 = Gumbel copula, 5 = Frank copula and 6 = Joe copula, 7 = Joe BB1 copula[1].The Gaussian copula is symmetric and has no tail dependence (Aloui et al., 2013). The Student-*t* and Frank copulas describe situations of extreme symmetric tail dependence andtail independence, respectively, while the Gumbel copula describes situations ofasymmetric

[1]We are thankful to the anonymous referee for suggesting to include the Joe BB1 copula

tail dependence (Rodriguez, 2007). Further, the Clayton copula exhibits greater dependence in the negative tail than in the positive, whereas the Gumbel copula exhibits greater dependence in the upper tail than in the lower tail (Aloui et al., 2013). Joe copula captures the positive tail dependence. Joe BB1 copula provides both left and tail dependence.

4.2.R-Vine Copulas

4.2.1. The Pre-GFC Period

The trees for the pre-GFC period are shown in Figures 1–3. It can be observed that the R-vine copula structure is more flexible than the C-vine structure (shown in appendix). Tree-1 shows two categories of commodities. On one hand, the agriculture commodities are linked together, that is, wheat, oats, corn and soybeans, while on the other hand, the precious metals (gold, silver, PT and PD) and the energy commodities (WTI, HO and NG) are separately linked to each other.

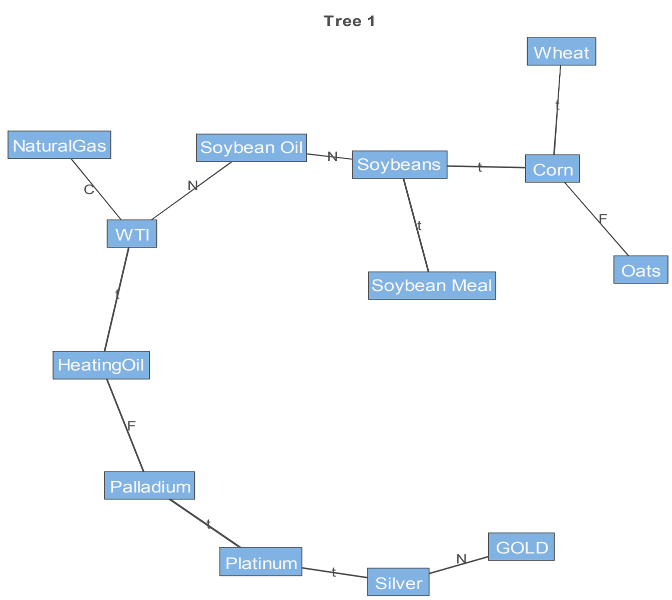


Figure 1. R-vine tree-1 pre-GFC

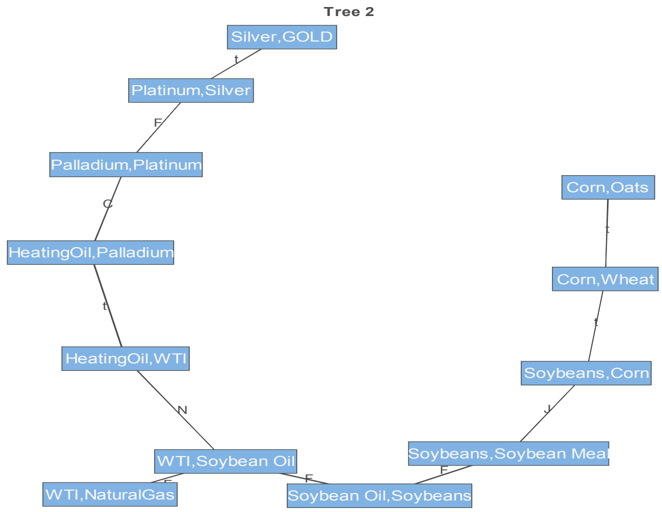


Figure 2. R-vine tree-2 pre-GFC

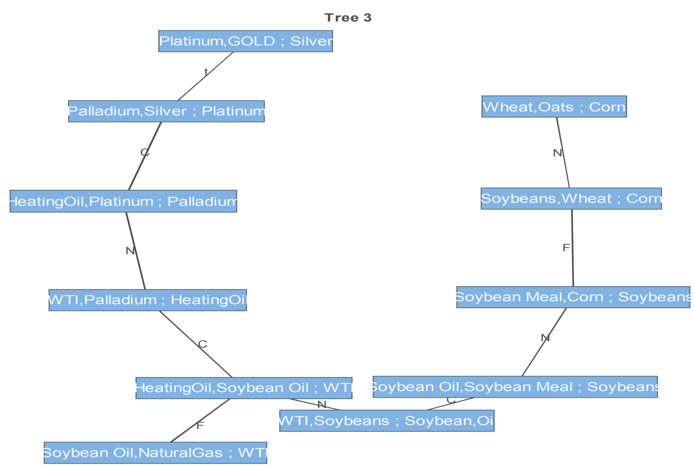


Figure 3. R-vine tree-3 pre-GFC

This phenomenon is also apparent in Tables 2 and 3. Table 3 shows the types and number of copulas integral in our analysis. The advantage of using the R-vines in terms of capturing the complex patterns of dependency can be seen in Table 3. It is apparent that at different dependencies, conditioned across the same node, various copulas are used. For instance, in column 1, the first copula used is the Student-*t* copula (no. 2), followed by the Frank copula (no. 5), then the Clayton copula (no.3)for a couple of levels, again the Frank copula (no. 5),the Gaussian copula (no.1), again the Frank copula (no. 5), then two cases of the Student-*t* copula (no 2), then the Gaussian copula again, followed by the Student-*t* copula (no. 2) and, finally, the Frank copula again (no. 5).

Table 2
Pre-GFC R-vine copula structure

	Corn	SO	SM	Oats	SB	Wheat	GOLD	Silver	PD	PT	WTI	HO	NG
Corn	4												
SO	7	6											
SM	8	7	1										
Oats	10	8	7	3									
SB	9	10	8	7	5								
Wheat	13	9	10	8	7	7							
GOLD	12	13	9	10	8	13	8						
Silver	11	12	13	9	10	2	13	10					
PD	2	11	12	13	9	11	2	13	13				
PT	3	2	11	12	13	12	11	2	9	2			
WTI	5	3	2	11	12	9	12	11	12	9	9		
HO	6	5	3	2	11	10	9	12	2	12	11	11	
NG	1	1	5	5	2	8	10	9	11	11	12	12	12

Note: HO, NG, PD, PT, SM, SO and SB stand for heating oil, natural gas, palladium, platinum, soybean meal, soybean oil and soybeans, respectively. The data period is from2 January 2003 to 19 December 2016.

Table 3
Pre-GFC R-vine copula specification matrix

	Corn	SO	SM	Oats	SB	Wheat	GOLD	Silver	PD	PT	WTI	HO	NG
Corn													
SO	2												
SM	5	5											
Oats	3	1	3										
SB	3	1	5	5									
Wheat	5	6	1	5	1								
GOLD	1	1	5	1	5	5							
Silver	5	5	1	5	5	1	5						
PD	2	5	5	1	5	1	3	1					
PT	2	5	5	5	1	1	5	1	1				
WTI	1	5	1	3	1	2	3	1	5	3			
HO	2	2	6	5	5	2	5	3	5	1	2		
NG	5	2	2	2	1	1	2	2	3	1	5	2	

Note: HO, NG, PD, PT, SM, SO and SB stand for heating oil, natural gas, palladium, platinum, soybean meal, soybean oil and soybeans, respectively. The data period is from2 January 2003 to 19 December 2016.

This mixture of usage is noticeable across Table 3 at various levels in the tree structure used to capture dependencies. The bottom row consists primarily of Student-*t* copulas. The R-vine copulas used to capture the co-dependencies are dissimilar to the pre-GFC period C-vine copulas. In Appendix 2, 15 Gaussian copulas, 14 Student-*t* copulas, 12 Clayton copulas, 1 each of Gumbel, Joe and Joe BB1 copulas, and 34 Frank copulas were used. However, in Table 3, 24 Gaussian, 14 Student-*t*, 9 Clayton, 29 Frank and 2 Joe copulas are used, with no instances of Gumbel and Joe BB1 copulas. This is possible because different co-dependencies are easily captured in the tree without any constraints on the pairings in the R-vine copulas.

To conserve space, we have not reported the details of the parameters estimated, but the tau matrix for R-vines is shown in Table 4. The results in Table 4 for R-vines are different from those reported in Appendix 3 for C-vines. The strongest dependencies between various commodities are captured by the values reported at the bottom of the table. Overall, the picture of dependencies is similar to those captured by the C-vine analysis.

Table 4
Pre-GFC R-vine copula tau matrix

	Corn	SO	SM	Oats	SB	Wheat	GOLD	Silver	PD	PT	WTI	HO	NG
Corn													
SO	-0.03												
SM	0.01	-0.02											
Oats	0.03	-0.01	0.05										
SB	0.02	0.04	-0.04	-0.02									
Wheat	-0.02	0.01	-0.04	-0.02	-0.08								
GOLD	0.01	-0.01	-0.01	0.02	0.01	-0.02							
Silver	0.02	-0.03	-0.01	-0.01	0.04	-0.06	-0.02						
PD	-0.01	-0.04	-0.01	-0.03	0.01	-0.04	0.06	0.02					
PT	-0.03	-0.03	0.03	-0.01	-0.02	-0.10	-0.06	0.05	0.04				
WTI	-0.02	0.02	0.06	0.05	0.05	-0.12	0.03	0.02	0.02	0.07			
HO	0.07	-0.04	0.07	-0.40	0.03	0.18	0.06	0.05	0.05	0.05	0.02		
NG	0.28	0.46	0.39	0.60	0.36	0.60	0.54	0.39	0.11	0.23	0.18	0.66	

Note: HO, NG, PD, PT, SM, SO and SB stand for heating oil, natural gas, palladium, platinum, soybean meal, soybean oil and soybeans, respectively. The data period is fromJanuary 2, 2003 to December 19, 2016.

4.2.2. The GFC Period

Figures 4 and 5 illustrate the trees for the R-vine copulas during the GFC period. Figure 4 shows that the dependencies have changed because of the influence of the GFC, and SO and HO act as the central commodities through which all three categories of commodities are linked.

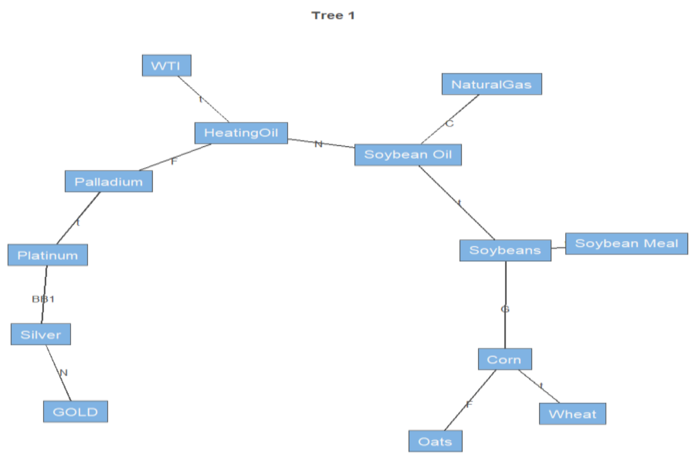


Figure 4. R-vine tree-1 GFC

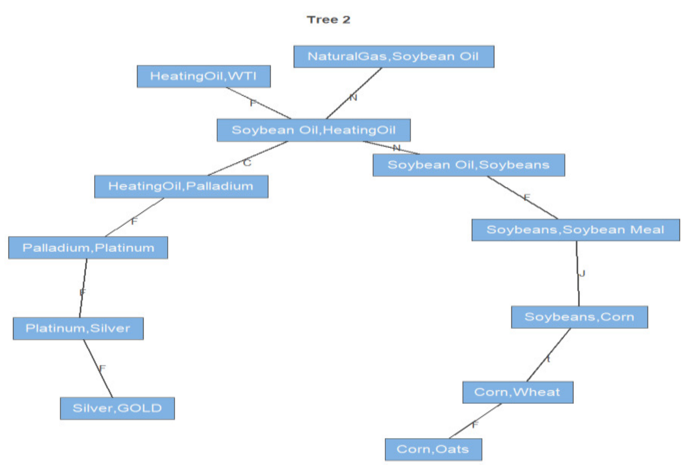


Figure 5. R-vine tree-2 GFC

For instance, all precious metals and the energy commodities are linked to HO, which in turn is connected to SO; all agricultural commodities are linked to soybeans, which is linked to SO. However, in the pre-GFC period the links between agricultural and precious metals and energy commodities were not clear. Table 5 again implies the significance of capturing tail risk in financial and economic downturns leading to fat-tailed distributions. The number of Gaussian copulas is 19, whereas the Student-*t* copula leads in the table, appearing 31times. There are 11 appearances of the Frank copula, 9of the Clayton, 3 of the Gumbel and 4 of the Joe copula, whereas the Joe BB1 copula appears only in one instance.

Table 5
GFC R-vine copula specification matrix

	Corn	SO	SM	Oats	SB	Wheat	GOLD	Silver	PD	PT	WTI	HO	NG
Corn													
SO	2												
SM	5	1											
Oats	2	2	3										
SB	5	6	2	5									
Wheat	2	6	1	2	1								
GOLD	2	2	1	1	5	1							
Silver	5	1	2	1	4	2	5						
PD	2	5	5	2	2	1	2	4					
PT	2	1	3	2	1	1	3	6	2				
WTI	3	2	1	3	2	2	3	1	1	3			
HO	2	2	6	2	1	2	2	5	3	2	1		
NG	5	2	4	2	2	1	7	2	5	2	1	3	

Note: HO, NG, PD, PT, SM, SO and SB stand for heating oil, natural gas, palladium, platinum, soybean meal, soybean oil and soybeans, respectively. The data period is from2 January 2003 to 19 December 2016.

Table 6
GFC R-vine copula tau matrix

	Corn	SO	SM	Oats	SB	Wheat	GOLD	Silver	PD	PT	WTI	HO	NG
Corn													
SO	-0.02												
SM	-0.01	-0.02											
Oats	-0.01	0.01	0.06										
SB	-0.01	0.01	-0.03	-0.03									
Wheat	-0.04	0.01	-0.03	-0.02	-0.09								
GOLD	0.01	0.01	0.04	0.01	0.00	-0.03							
Silver	0.01	-0.02	0.01	0.02	0.03	-0.01	-0.03						
PD	-0.04	-0.01	0.02	-0.02	0.01	-0.07	-0.03	0.02					
PT	-0.03	-0.01	0.02	0.03	0.00	-0.06	0.06	0.02	0.01				
WTI	0.04	0.04	0.08	0.02	-0.03	-0.11	0.03	0.06	-0.04	0.03			
HO	0.10	-0.04	0.07	-0.38	0.07	0.20	0.04	0.07	0.08	0.06	0.06		
NG	0.38	0.47	0.44	0.68	0.34	0.56	0.53	0.40	0.18	0.64	0.23	0.10	

Note: HO, NG, PD, PT, SM, SO and SB stand for heating oil, natural gas, palladium, platinum, soybean meal, soybean oil and soybeans, respectively. The data period is from January 2, 2003 to December 19, 2016.

Table 6 presents the results of the tau matrix for the GFC period. The varying dependencies in the R-vine analysis during the GFC provide us with some important observations. Regarding the values at the bottom of Table 6, 8 dependencies out of 12 indicate an increase, compared with the pre-GFC entries in Table 4. Further, the table reveals that the number of negative entries is 28, less than the 32 negative observations in Table 4. Overall, results in Table 6 indicate that dependencies between these major commodities have increased during the crisis period.

4.2.3. The Post-GFC Period

In this subsection, we provide a detailed analysis of the post-GFC R-vine copula structure. Figure 6showsthat the relationships between the commodities markets have changed in the post-GFC period compared to the GFC period.

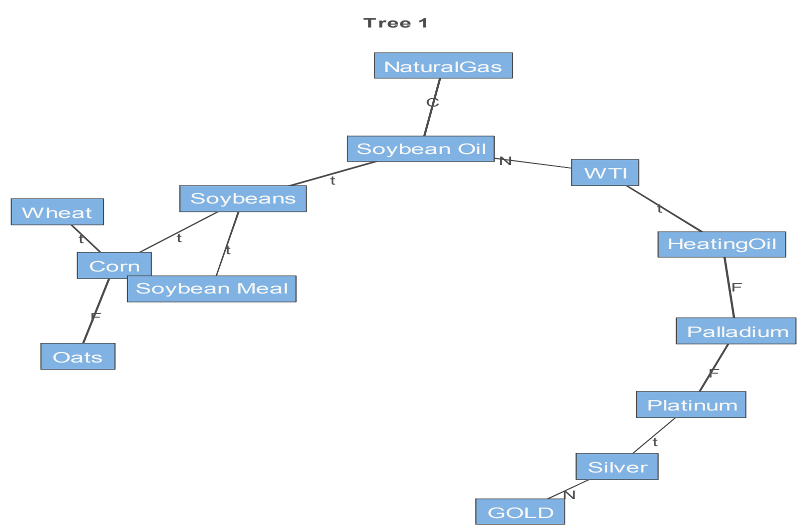


Figure 6. R-vine tree-1 post-GFC

It can be seen in tree-1 that SO acts as a central commodity and that precious metals are linked to the agricultural commodities through WTI, which is similar to the pre-GFC linkages to some extent. Table 7 shows the various categories of copulas used to capture the dependencies in the post-GFC period. The Gaussian copula appears 19 times, while the Student-*t* copula yet again dominates, being used on 25 occasions in Table 7, a considerable reduction from 31 times during the GFC period. The Clayton copula is used on 8 occasions, the Gumbel on two. The Frank copula appears 20 times, the Joe copula appears 4 times, and finally the Joe BB1 copula does not appear in the Table.

Table 7
Post-GFC R-vine copula specification matrix

	Corn	SO	SM	Oats	SB	Wheat	GOLD	Silver	PD	PT	WTI	HO	NG
Corn													
SO	1												
SM	2	2											
Oats	1	1	5										
SB	3	2	6	5									
Wheat	2	1	5	2	1								
GOLD	1	2	2	1	5	2							
Silver	2	5	1	1	6	5	5						
PD	1	1	5	5	3	6	3	1					
PT	2	2	1	4	5	5	1	2	1				
WTI	2	3	2	2	5	1	3	4	5	3			
HO	5	2	1	6	5	5	5	1	3	5	2		
NG	1	2	2	2	2	2	2	5	3	2	5	2	

Note: HO, NG, PD, PT, SM, SO and SB stand for heating oil, natural gas, palladium, platinum, soybean meal, soybean oil and soybeans, respectively. The data period is from2 January 2003 to 19 December 2016.

The tau dependency matrix presented in Table 8 shows that the dependencies have changed significantly in the post-GFC period. Compared to the GFC period, the dependencies in the bottom row have reduced in 6 of the 12 total cases in the post-GFC period. Table 8 reports 28 cases of negative relationships which is same as shown in Table 6 for the GFC period.

Though these changes are appealing, they are unable to provide an indication of the usefulness of R-vine modelling. In the next section, we empirically examine the VaR, which provides details of its use in risk assessment. In particular, our analysis offers important practical implications in that it helps examine the risk characteristics of commodity portfolios and changes in their conditional dependence structure across three financial periods, incorporating the GFC. A precise estimation and interpretation of the dependence structure among these major commodities offer significant practical implications to investors and policymakers alike. Our paper provides new insights which could be useful in developing risk strategies for investment and hedging purposes, especially during more volatile periods in the markets.

Table 8
Post-GFC R-vine copula tau matrix

	Corn	SO	SM	Oats	SB	Wheat	GOLD	Silver	PD	PT	WTI	HO	NG
Corn													
SO	0.01												
SM	-0.02	-0.03											
Oats	-0.01	-0.01	0.01										
SB	0.06	-0.01	0.01	0.01									
Wheat	-0.03	0.00	0.02	-0.05	-0.02								
GOLD	-0.08	0.02	-0.01	-0.02	-0.02	-0.01							
Silver	-0.06	0.01	0.01	0.04	0.00	0.00	-0.03						
PD	-0.04	-0.04	-0.02	-0.01	0.03	0.03	0.07	0.02					
PT	-0.07	-0.02	-0.02	0.01	-0.01	0.02	-0.03	0.06	-0.04				
WTI	-0.12	0.03	0.04	0.08	0.05	0.04	0.03	0.01	0.01	0.08			
HO	0.20	0.09	-0.05	0.08	-0.38	0.05	0.05	0.07	0.08	0.07	0.01		
NG	0.59	0.28	0.46	0.38	0.58	0.34	0.53	0.41	0.10	0.23	0.18	0.65	

Note: HO, NG, PD, PT, SM, SO and SB stand for heating oil, natural gas, palladium, platinum, soybean meal, soybean oil and soybeans, respectively. The data period is from2 January 2003 to 19 December 2016.

Fink et al. (2017) apply Markov-switching R-vine models to investigate the existence of different global dependence regimes. In particular, they identify times of normal and abnormal states within a dataset consisting of North American, European and Asian indices. However, our work is different from theirs in that we choose a specific time period to signify the GFC, whereas they use smoothed rolling windows in a Markov-switching analysis. There are further limited studies which apply the same approach as we do. For instance, Beil (2013) uses vine copulas to understand the global stock indices. Further, Allen et al. (2017) also apply regular vine copulas to understand the dependencies in 10 major European stock markets by breaking the sample into pre-crisis, crisis and post-crisis periods. However, Beil (2013) does not make use of the Gumbel copula to a great extent. Moreover, in Allen et al. (2017), the sample used is different compared to ours.

4.3.Economic Significance – VaR Analysis

We apply C-vine and R-vine copulas to capture the dependence structure among some of the key commodity markets which, in turn, could be crucial for portfolio evaluation and risk modelling. The R-vine approach provides better results than the standard bivariate copula frame work since the copulas selected via the vine copulas framework are more sensitive to the asset's return distributions. The co-dependencies computed using R-vine copulas are useful for portfolio VaR analysis. For all three subsamples (i.e., pre-GFC, GFC and post-GFC), we create an equally weighted portfolio of the 13commodities to explore the importance of vine copulas in modelling VaR[2]. The 13 selected assets in the portfolio are our 13 commodities. We employ a 250-day moving window framework to forecast the VaR for this equally weighted portfolio.[3]

The main steps of our analysis are illustrated below:

1. Convert the data for commodities into log returns and select a 250-daymoving window of returns.
2. Apply GARCH (1,1) with Student-*t* innovations to convert the log returns into an IID series. The same model is fitted in all the iterations to maintain uniformity in the approach, which also makes the analysis a little less intensive.
3. Take the residuals from Step2 and standardise them with the deviations obtained in Step 2.
4. Convert these residuals to Student-*t* marginals for the estimation of copulas. These steps are repeated for all the 13commodities to obtain a multivariate matrix of uniform marginals.

---

[2]In the paper, we report the results for the full sample only. Please refer to the Appendix for the sub-sample results.
[3] To analyse the sensitivity of the results with respect to the rolling window size we also chose a 500-day rolling window and found that the results were robust to the window size selected. Though these results are not presented, they are available upon request.

5. Fit an R-vine to the multivariate data so obtained and generate simulations using the fitted R-vine model. We generate 1000 simulations per commodity for forecasting a day ahead VaR.
6. Convert the simulated uniform marginals to standardised residuals and simulate returns using GARCH simulations.
7. Generate a series of simulated daily portfolio returns to forecast 1% and 5% VaR.
8. Repeat Steps 1 to 7 for a moving window.

The above approach leads to VaR forecasts, which, not being time-dependent, have the advantage of being co-dependent on the commodities in the portfolio. We use this framework as a manifestation of a practical application of the co-dependencies captured by the flexible vine copula approach applied to construct VaR forecasts. Figure 7-9 plot the 1% and 5% VaR forecasts along with the original portfolio return series obtained bythe method for the full sample. The plot shows that the VaR forecasts closely follow the daily returns with few violations.

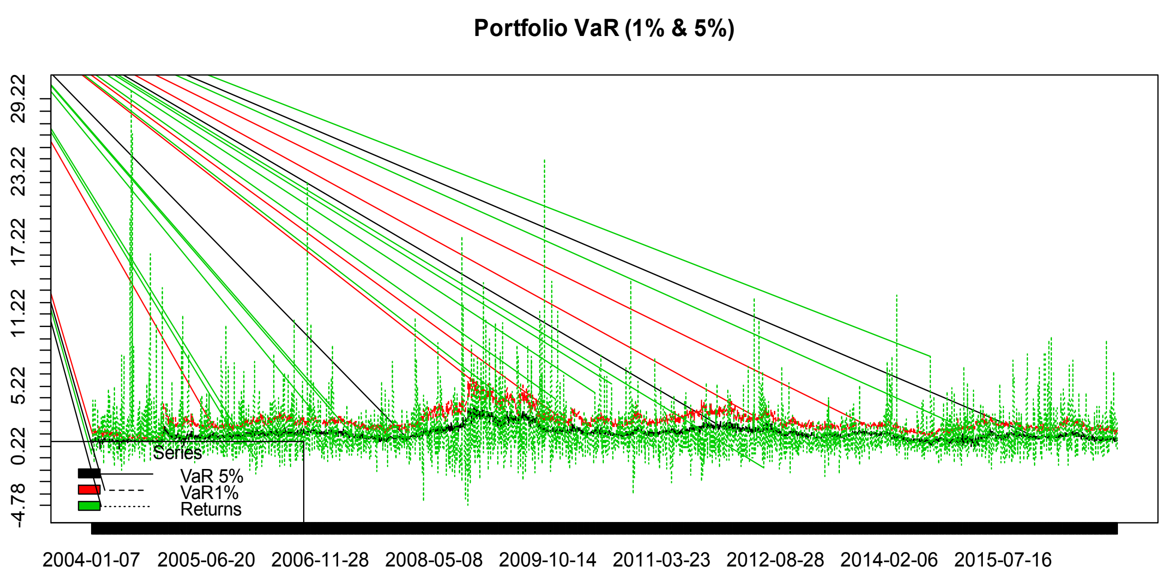


Figure 7. Portfolio VaR analysis based on the application of vine copulas

For a better comparison of the performance of our vine copula VaR forecasts, we also use our series of commodity returns combined into an equally weighted portfolio to construct a simulation of a VaR analysis based on the use of a GARCH(1,1) model[4]. The relative number of violations of the VaR set at 1% and 5% indicate whether our vine copula approach better captures the complex structure of dependencies and is better suited to VaR analysis.

This analysis is outlined as follows:

1. Convert the data sample to log returns and select a moving window of 250 returns.[5]
2. Fit the GARCH(1,1) with Normal innovations to convert the log returns into an IID series.
3. Extract the fit from Step 2 and simulate 1000 returns per asset.
4. Repeat Steps 2 and 3 for all commodities followed by the computation of the portfolio return from the simulated series.
5. Generate a series of simulated daily portfolio returns to forecast 1% and 5% VaR.
6. Repeat Steps 1 to 5 for a moving window.

Figure 8 plots the 1% and 5% VaR forecasts along with original portfolio return series obtained from the use of the GARCH (1, 1) model. A close examination of the plots reveals that the use of the GARCH (1,1) model leads to multiple violations of the VaR 5% (black line) and VaR 1% (red line) compared to the VaR forecasts based on the application of vine copulas. Further, the results presented in the appendix for sub-sample analysis also provide robustness to the results shown in Figures 7A and 8A for the full sample. Overall, our vine copula models are best suited over the GRACH (1,1) model to compute the portfolio VaR during the considered time period.

---

[4]We have reported the results for the full sample only. The readers may kindly refer to the sub-sample results presented in the appendix.

[5]To analyse the sensitivity of the results with respect to the rolling window size we also chose 500-day rolling window and found that the results were robust to the window size selected. Though these results are not presented, they are available upon request.

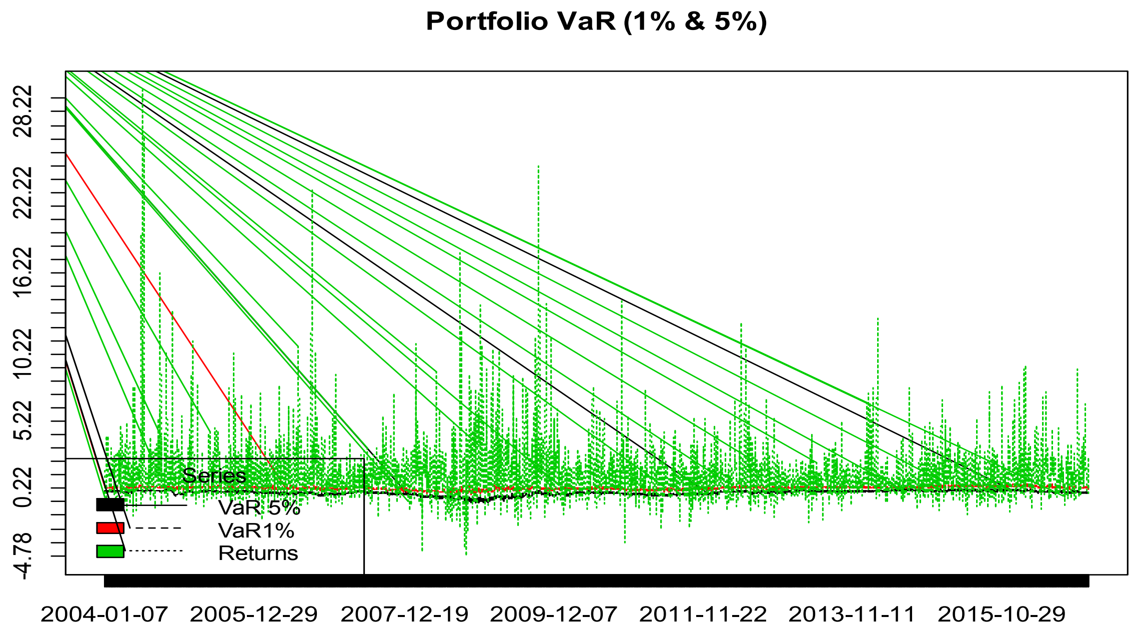


Figure 8. Portfolio VaR analysis based on the application of GARCH (1,1) model

4.4. Analysis of efficient portfolios

To illustrate the different combinations of the commodities, Figure 9 exhibits the efficient frontier of the portfolios. We plot the efficient frontier under the short-selling constraints such that NG and gold are located on the efficient frontier, representing the lowest and the highest expected returns among all the commodities considered, respectively[6].

The maximum Sharpe ratio and maximum utility can be achieved on the efficient frontier in Figure 9 through rational asset allocation across the commodities. In Figure 10, we remove the short-selling constraints and plot the efficient frontier again. The figure shows that none of the commodities now lie on the frontier.[7]

[6] We would like the readers to note that while performing the analysis of efficient portfolios with and without short-sales, the portfolios are not equally weighted.

[7] The analysis of efficient portfolios is based on in-sample results only as we estimate the mean and covariance matrices based on the entire sample, which is infeasible for real investments. Further, we have attempted to do the same analysis on simulated data from the GARCH and copula models, but we faced serious convergence issues.

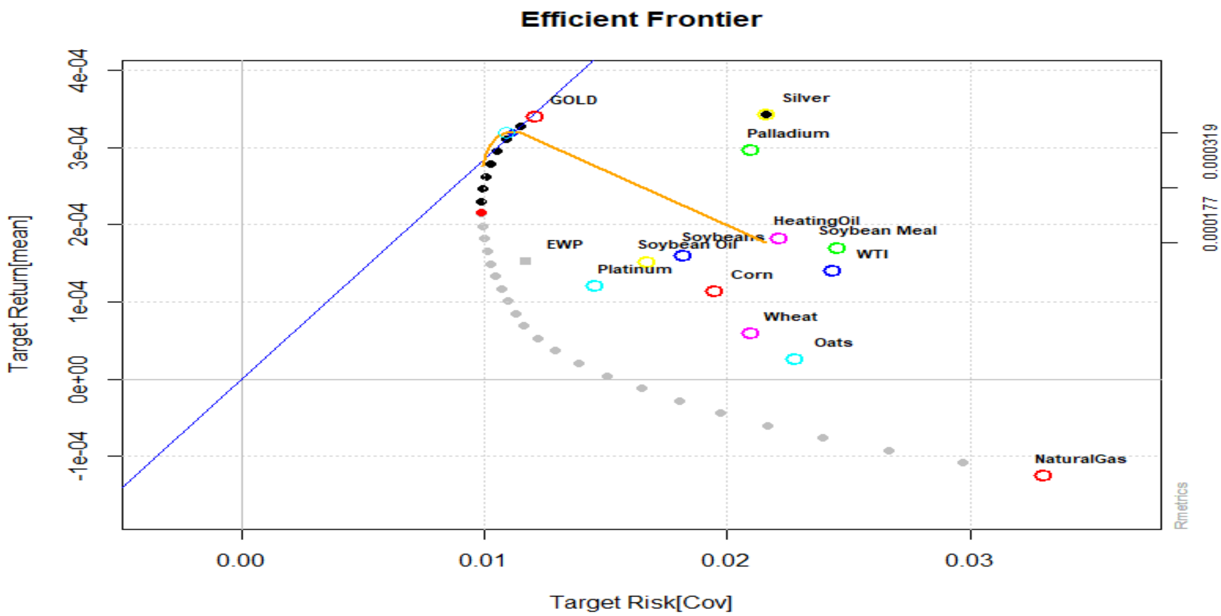


Figure 9. Efficient frontier of commodities' portfolio

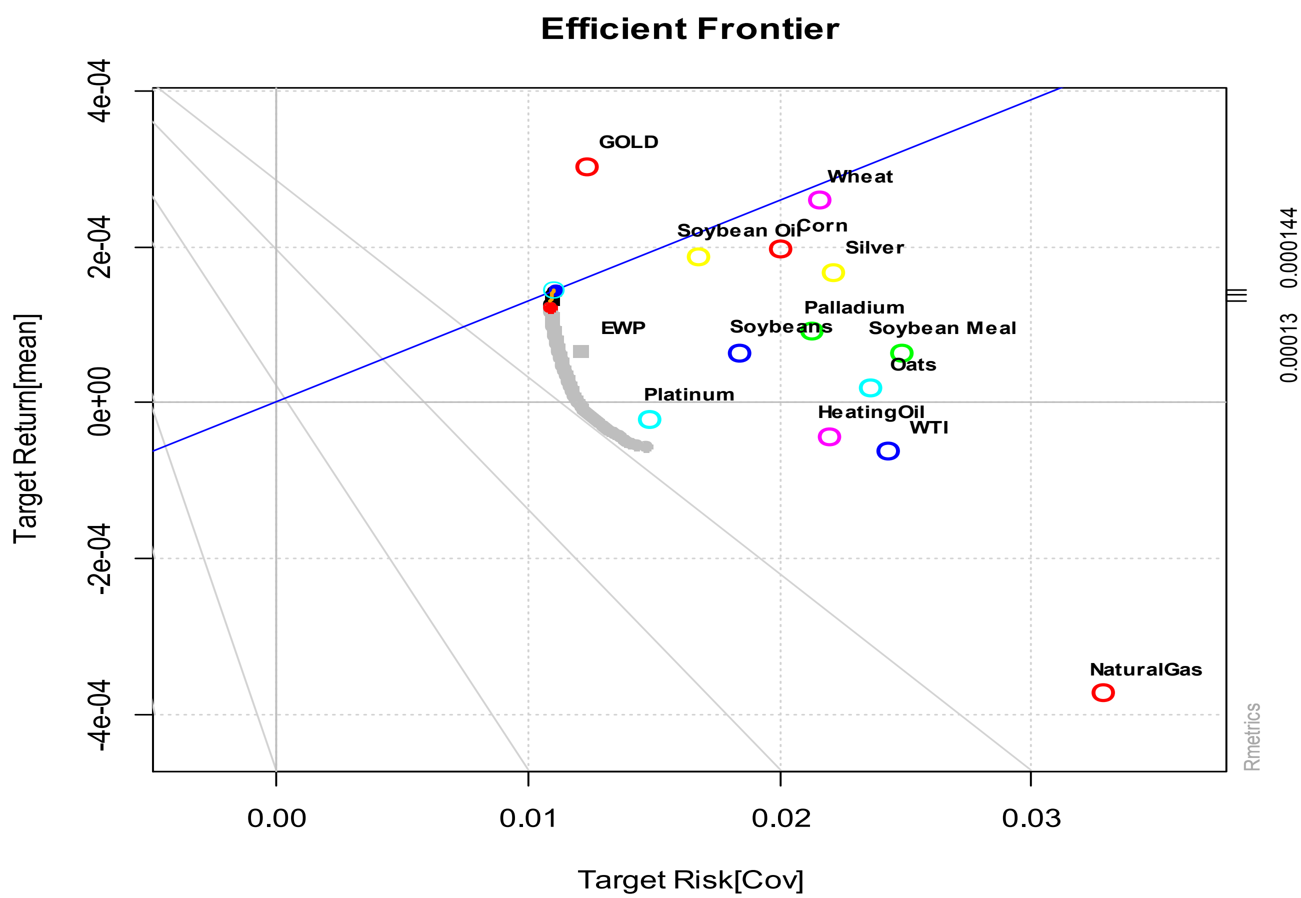
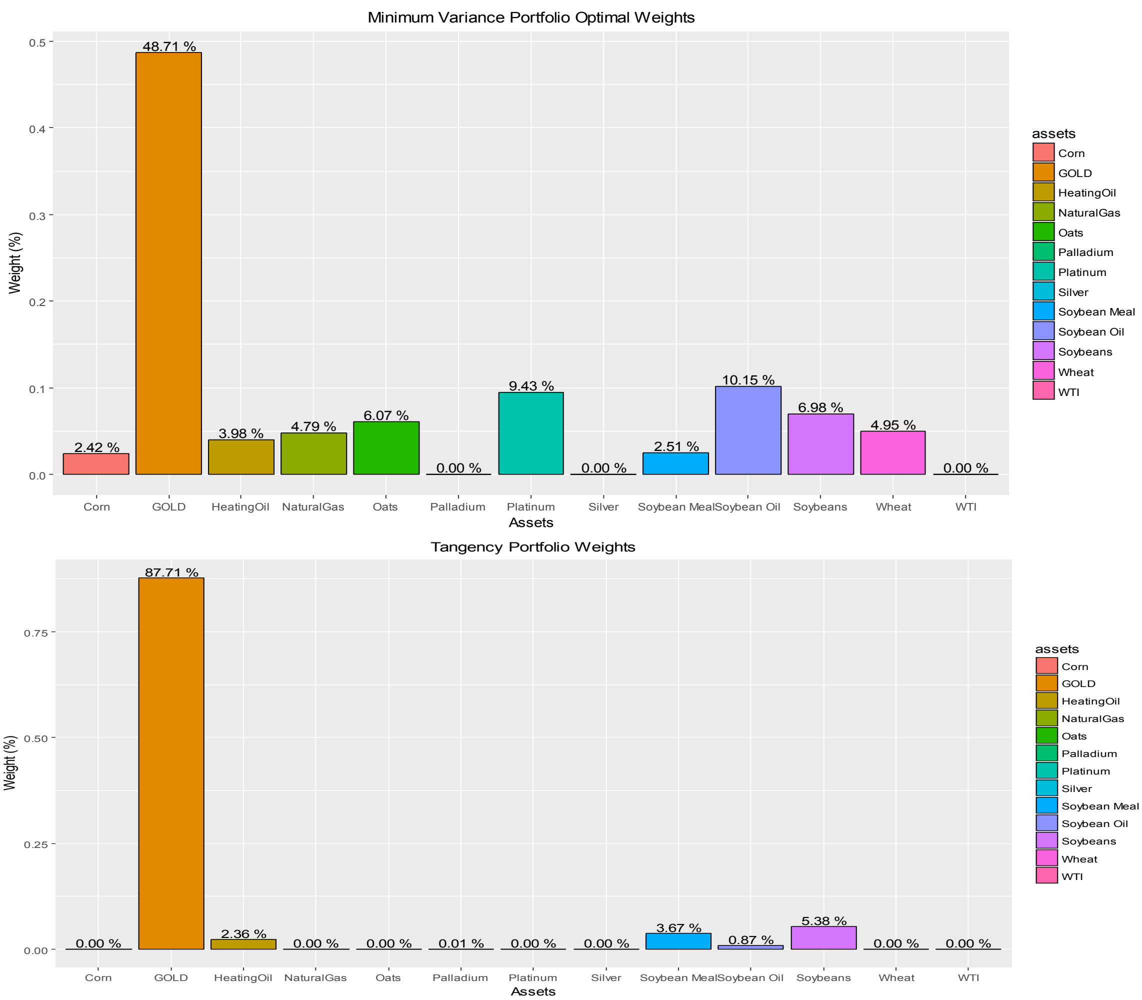


Figure 10. Efficient frontier of commodities' portfolio when short sales are allowed

Figure 11 illustrates different scenarios of the minimum variance portfolio and tangency portfolio. For example, the upper panel of Figure 11 shows that the minimum variance portfolio comprises the largest proportion of gold (48.71%), followed by SO (10.15%) and PT (9.43%). The portfolio has no proportion of PD, silver and WTI since their weights are zero. The middle panel of Figure 11 shows the results for the tangency portfolio weights where the capital market line touches the market portfolio. We observe that the proportion of gold in the portfolio increases to 87.71%, followed by soybeans (5.38%) and SM (3.67%) under the restriction on short sales.



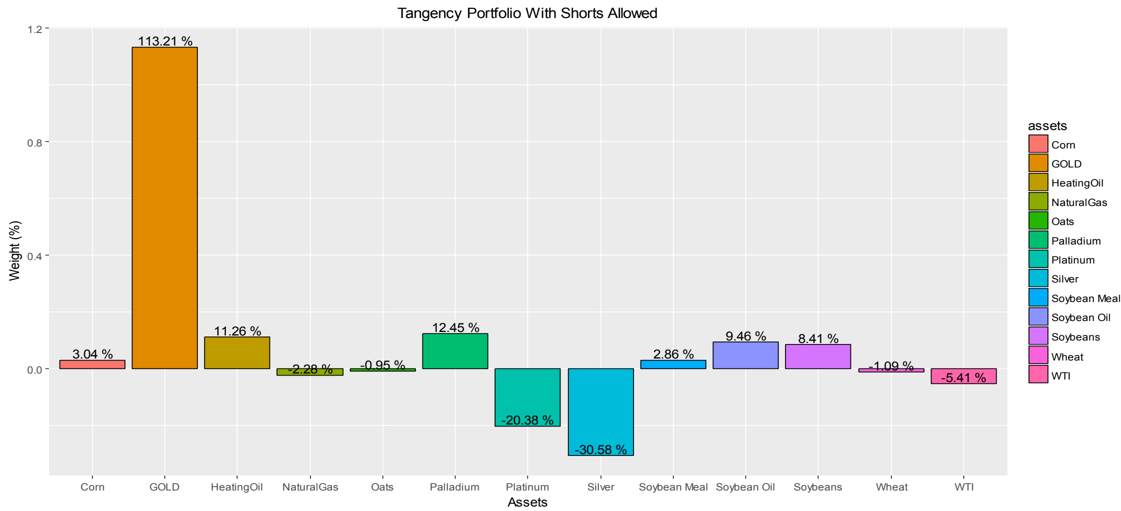


Figure 11. Portfolio weights under different scenarios

When we allow for short sales, in the lower panel of Figure 11, the proportion of gold shoots up to 113.21%, while for silver and PT, it goes down to as low as -30.58% and -20.38%, respectively.

Overall, the results for the minimum variance portfolio and tangency portfolio under both short sales and no short sales confirm that gold is the most attractive commodity among the investments.

5. Conclusion

This paper provides an application of the recently developed pair vine copula models to examine the changes in the co-dependencies of 13 major commodity markets from 2 January 2003 to 19 December 2016, spanning three periods of financial conditions (pre-GFC, GFC and post-GFC). We extend the extant literature by offering an in-depth analysis of the portfolio's multivariate dependence structure and risk dynamics by means of a copula counting technique. The results indicate that the dependency structure change in a complicated manner and further suggest that the Student-t copula appears the maximum number of times, especially during the GFC, highlighting the importance of fatter tails during financial turmoil.

Next, using the flexibility of this approach in financial risk modelling, we compute the portfolio VaR based on the dependencies obtained. The results show that the VaR forecasts closely follow the daily returns with few violations. These results are in contrast with those obtained from the same data using the GARCH (1,1) model and Gaussian distribution, and we conclude that our vine copula models are best suited to compute the portfolio VaR during the considered time period. We further plot the efficient frontier, construct the portfolio weights and show that gold is the most attractive commodity among the investments under all scenarios.

We contribute to the literature by identifying the risk characteristics of commodity portfolios and changes in their conditional dependence structure across three financial periods, incorporating the GFC. Overall, a precise estimation and interpretation of the dependence structure among these major commodities offer significant practical implications to investors and policymakers alike. Specifically, our approach provides new insights which could be useful in developing dependence-and investment-risk strategies for investment and hedging purposes, especially during more volatile periods in the markets.

Acknowledgement

The corresponding author thanks for supports from the National Natural Science Foundation of China under Grant No. 71774152, No. 91546109 and Youth Innovation Promotion Association of Chinese Academy of Sciences (Grant: Y7X0231505) are acknowledged.

References

Aas, K., Czado, C., Frigessi, A., Bakken, H., 2006. Pair-Copula Constructions of Multiple Dependence. Technical University of Munich.

Allen, D.E., McAleer, M., and Singh, A., 2017. Risk Measurement and Risk Modelling Using Applications of Vine Copulas. Sustainability, 9, 1-34.

Aloui, R., Aïssa, M. S. B., & Nguyen, D. K. (2013). Conditional dependence structure between oil prices and exchange rates: a copula-GARCH approach. Journal of International Money and Finance, 32, 719-738.

Antonakakis, N., Chang, T., Cunado, J., Gupta, R., 2018. The relationship between commodity markets and commodity mutual funds: A wavelet-based analysis. Finance Research Letters, 24, 1-9.

Bedford, T. Cooke, R.M., 2002. Vines—A new graphical model for dependent random variables. Annal Statistics, 30, 1031–1068.

Bouri, E., Gupta, R., Lahiani, A., Shahbaz, M., 2018. Testing for asymmetric nonlinear short- and long-run relationships between bitcoin, aggregate commodity and gold prices. Resources Policy, 57, 224-235.

Brechmann, E.C., 2010. Truncated and Simplified Regular Vines and their Applications. Center of Matemathics, Technical University of Munich.

Brechmann, E. C., & Czado, C. (2013). Risk management with high-dimensional vine copulas: An analysis of the Euro Stoxx 50. Statistics & Risk Modeling, 30(4), 307-342.

Dissman, J.F., 2010. Statistical Inference for Regular Vines and Application. Master's Thesis, TechnischeUniversitatMünchen, München, Germany.

Fernandez-Diaz, J.M., Morley, B., 2018. Interdependence among agricultural commodity markets, macroeconomic factors, crude oil and commodity index. Research in International Business and Finance. https://doi.org/10.1016/j.ribaf.2018.07.009.

Fernandez-Perez, A., Frijns, B., Tourani-Rad, A., 2016. Contemporaneous interactions among fuel, biofuel and agricultural commodities. Energy Economics, 58, 1-10.

Frechet, M., 1957. Les tableaux de corr´elationdont les marges et des bornessontdonn´ees. Annales de l'Universit´e de Lyon, Sciences Math´ematiquesetAstronomie, 20, 13–31.

Genest, C., Favre, A.-C., 2007. Everything you always wanted to know about copulas modeling but were afraid to ask. Journal of Hydrologic Engineering, 12 (4), 347–368

Genest, C., Ghoudi, K., Rivest, L.-P., 1995. A semiparametric estimation procedure of dependence parameters in multivariate families of distributions. Biometrika 82 (3), 543–552.

Genest, C., Rémillard, B., Beaudoin, D., 2009. Goodness-of-fit tests for copulas: a review and a power study. Insurance: Mathematics and Economics, 44, 199–213

Heinen, A., & Valdesogo, A. (2009). Asymmetric CAPM dependence for large dimensions: the canonical vine autoregressive model (No. Universidad Carlos III de Madrid). CORE, 1-30.

Hernandez, J.A. (2014). Are oil and gas stocks from the Australian market riskier than coal and uranium stocks? Dependence risk analysis and portfolio optimization. Energy Economics, 45, 528-536.

Hernandez, J.A. (2015). Vine copula modelling of dependence and portfolio optimization with application to mining and energy stock return series from the Australian market. Doctoral Dissertation, Edith CowanUniversity, Australia.

Ji, Q., Bour, E., Roubaud, D., Shahzad, S.J.H., 2018b. Risk spillover between energy and agricultural commodity markets: A dependence-switching CoVaR-copula model. Energy Economics. https://doi.org/10.1016/j.eneco.2018.08.015

Ji, Q., Bouri, E., Roubaud, D., 2018a. Dynamic network of implied volatility transmission among US equities, strategic commodities, and BRICS equities. International Review of Financial Analysis, 57, 1-12.

Ji, Q., Fan, Y., 2012. How does oil price volatility affect non-energy commodity markets? Applied Energy, 89(1), 273-280.

Ji, Q., Fan, Y., 2016a. Evolution of the world crude oil market integration: A graph theory analysis. Energy Economics, 53, 90-100.

Ji, Q., Fan, Y., 2016b. How do China's oil markets affect other commodity markets both domestically and internationally? Finance Research Letters, 19, 247-254.

Ji, Q., Liu, B., Fan, Y., 2018c. Risk dependence of CoVaR and structural change between oil prices and exchange rates: A time-varying copula model. Energy Economics. https://doi.org/10.1016/j.eneco.2018.07.012

Joe, H., 1997. Multivariate Models and Dependence Concepts; Chapman & Hall: London, UK, 1997

Kang, S.H., Mclver, R., Yoon, S., 2017. Dynamic spillover effects among crude oil, precious metal, and agricultural commodity futures markets. Energy Economics, 62, 19-32.

Lahmiri, S., Uddin, G.S., Bekiros, S., 2017. Clustering of short and long-term co-movements in international financial and commodity markets in wavelet domain. Physica A: Statistical Mechanics and its Applications, 486, 947-955.

Lebrun, R., Dutfoy, A., 2009a. An innovating analysis of the Nataf transformation from the copula view point. Probabilistic Engineering Mechanics 24, 312–320.

Lebrun, R., Dutfoy, A., 2009b. Do Rosenblatt and Natafiso probabilistic transformations really differ? Probabilistic Engineering Mechanics, 24, 577–584

Liu, B., Ji, Q., Fan, Y., 2017. A new time-varying optimal copula model identifying the dependence across markets. Quantitative Finance, 17(3), 437-453.

Mensi, W., Hammoudeh, S., Nguyen, D.K., Yoon, S., 2014. Dynamic spillovers among major energy and cereal commodity prices. Energy Economics, 43, 225-243.

Mensi, W., Hkiri, B., AI-Yahyaee, K.H., Kang, S.Y., 2018. Analyzing time–frequency co-movements across gold and oil prices with BRICS stock markets: A VaR based on wavelet approach. International Review of Economics & Finance, 54, 74-102.

Nazlioglu, S., Soytas, U., 2012. Oil price, agricultural commodity prices, and the dollar: A panel cointegration and causality analysis. Energy Economics, 34, 1098-1104.

Nelsen, R., 2006. An Introduction to Copulas, 2nd ed.; Springer: New York, NY, USA.

Nicola, F., Pace, P., Hernandez, M.A., 2016. Co-movement of major energy, agricultural, and food commodity price returns: A time-series assessment. Energy Economics, 57, 28-41.

Rodriguez, J. C. (2007). Measuring financial contagion: A copula approach. Journal of Empirical Finance, 14(3), 401-423.

Shahzad, S.J.H., Hernandez, J.A., AI-Yahyaee, K.H., Jammazi, R., 2018. Asymmetric risk spillovers between oil and agricultural commodities. Energy Policy, 118, 182-198.

Sklar, A., 1996. Random variables, distribution functions, and copulas – a personal look backward and forward in Distributions with Fixed Marginals and Related Topics, ed. by L.

R¨uschendorff, B.Schweizer, and M. Taylor, pp. 1–14. Institute of Mathematical Statistics, Hayward, CA.

Tang, K., & Xiong, W. (2012). Index investment and the financialization of commodities. Financial Analysts Journal, 68(5), 54-74.

Vuong, Q.H., 1989. Likelihood ratio tests for model selection and non-nested hypotheses. Econometrica, 57, 307–333.